QUIZ 23 SOLUTIONS: LESSON 31
NOVEMBER 16, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Put the following augmented matrix into **reduced row-echelon** form:

$$\left[\begin{array}{cc|c} 2 & -3 & -8 \\ -1 & 4 & 9 \end{array}\right].$$

Label each row operation you use.

Solution: There are many ways to put this in reduced row-echelon form, I outline one way below:

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} -1 & 4 & 9 \\ 2 & -3 & -8 \end{array}\right] \xrightarrow{2R_1+R_2 \to R_2} \left[\begin{array}{cc|c} -1 & 4 & 9 \\ 0 & 5 & 10 \end{array}\right]$$

$$\xrightarrow{-R_1 \to R_1} \left[\begin{array}{cc|c} 1 & -4 & -9 \\ 0 & 5 & 10 \end{array}\right] \xrightarrow{R_2/5 \to R_2} \left[\begin{array}{cc|c} 1 & -4 & -9 \\ 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{4R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$$

2. [6 pts] Solve the following system of equations using any method:

$$\begin{cases} -x + 2y - 3z = -5 \\ x + y - z = 2 \\ x + 4z = 3 \end{cases}$$

Solution: There are many ways to solve this system of equations, I use the method of Guass-Jordan elimination. We write

$$\xrightarrow{\text{Translate}} \left[\begin{array}{ccc|c} -1 & 2 & -3 & -5 \\ 1 & 1 & -1 & 2 \\ 1 & 0 & 4 & 3 \end{array}\right] \xrightarrow{R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} -1 & 2 & -3 & -5 \\ 0 & 3 & -4 & -3 \\ 1 & 0 & 4 & 3 \end{array}\right]$$

$$\xrightarrow{R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} -1 & 2 & -3 & -5 \\ 0 & 3 & -4 & -3 \\ 0 & 2 & 1 & -2 \end{array}\right] \xrightarrow{-R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 0 & 3 & -4 & -3 \\ 0 & 2 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{-R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 0 & 1 & -5 & -1 \\ 0 & 2 & 1 & -2 \end{array}\right] \xrightarrow{-2R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 0 & 1 & -5 & -1 \\ 0 & 0 & 11 & 0 \end{array}\right]$$

$$\xrightarrow{R_3/11\to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 0 & 1 & -5 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{5R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{-3R_3+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{2R_3+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Thus, the solution is

$$(x, y, z) = (3, -1, 0).$$